Spiral cylindrique avec courbes terminales en arc de cercle

Influence d'une déformation des courbes terminales

Calculs numériques et approximations de Haag

Caractéristiques du spiral

Référence :C:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :C:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-12}$

Elinvar $\quad \rho_s = 8 \times 10^3\,kg \cdot m^{-3} \quad E = 1.7 \times 10^{11}\,Pa \quad G = 6.538 \times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s \cdot 360 \cdot deg \quad \psi_0 = 3.654 \times 10^3\,deg \quad L := R_0 \cdot \psi_0 \quad L = 318.872\,mm$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Référence :C:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha) \qquad W_{f3} := W_{f\_rect}(ép, ha)$

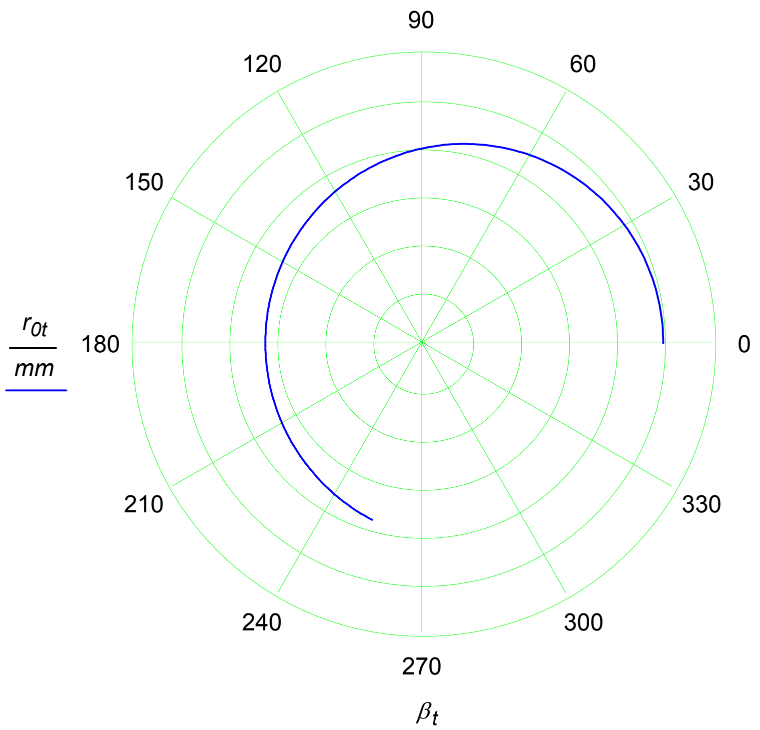
Courbe terminale

$$\beta := 121 \cdot deg \quad \beta_{Ph} := racine\left[\beta \cdot \left(\sqrt{2} \cdot sin(\beta) - 1\right) + sin(\beta) \cdot cos(\beta), \beta\right] \quad r_{Ph} := \frac{R_0}{\sqrt{2} \cdot sin(\beta_{Ph})} \quad l_t := r_{Ph} \cdot 2 \cdot \beta_{Ph}$$

$$\beta_0(r_t) := \frac{l_t}{2 \cdot r_t} \qquad X_{0t}(r_t, \alpha_t) := R_0 - r_t + r_t \cdot cos(\alpha_t) \qquad Y_{0t}(r_t, \alpha_t) := r_t \cdot sin(\alpha_t)$$

$$n_t := 201 \quad j := 0 .. n_t - 1 \quad \Delta\alpha_t := \frac{2 \cdot \beta_0(r_{Ph})}{n_t - 1} \quad \alpha_{t_j} := j \cdot \Delta\alpha_t \quad X_{t_j} := X_{0t}(r_{Ph}, \alpha_{t_j}) \quad Y_{t_j} := Y_{0t}(r_{Ph}, \alpha_{t_j})$$

$$r_{0t} := \overrightarrow{\sqrt{X_t^2 + Y_t^2}} \qquad \beta_t := \overrightarrow{Atan(X_t, Y_t)}$$



Perturbation de période en position horizontale

Cas de courbes de Phillips symétriques

$Z_{0t}(r_t,\alpha) := X_{0t}(r_t,\alpha) + i\cdot Y_{0t}(r_t,\alpha)$ $\quad$ $\mathbf{OA} := R_0\cdot e^{i\cdot\pi}$ $\quad$ $\mathbf{OB} := R_0\cdot e^{i\cdot(\pi+\psi_0)}$ $\quad$ $L_t := L + 2\cdot l_t$

$$Z_1(r_t) := \frac{1}{R_0^2}\cdot\int_0^{2\cdot\beta_0(r_t)} Z_{0t}(r_t,\alpha)\cdot r_t\,d\alpha - i \qquad \rho_{01} := |Z_1(r_{Ph})| \qquad \varphi_{01} := \arg(Z_1(r_{Ph}))$$

$\boxed{\rho_{01} = 0}$ $\quad$ $\boxed{\varphi_{01} = -24.386\,deg}$

$$Z_2(r_t) := \frac{1}{R_0^3}\cdot\int_0^{2\cdot\beta_0(r_t)} r_t\cdot\alpha\cdot Z_{0t}(r_t,\alpha)\cdot r_t\,d\alpha + 1 \qquad \rho_{02} := |Z_2(r_{Ph})| \qquad \varphi_{02} := \arg(Z_2(r_{Ph}))$$

$\boxed{\rho_{02} = 1.074}$ $\quad$ $\boxed{\varphi_{02} = 145.651\,deg}$

$$\mathbf{w_{aPh}}(\theta) := \frac{R_0^2}{L_t^2}\cdot\theta^2\cdot\rho_{02}\cdot\left(e^{-i\cdot\varphi_{02}}\cdot\mathbf{OA} - e^{i\cdot\varphi_{02}}\cdot\mathbf{OB}\cdot e^{i\cdot\theta}\right) \qquad \mathbf{w_{aPh}}(\theta_0) = 0.012 + 0.036i\,mm$$

$$H(x) := x\cdot(1+x^2)\cdot J1(x) - 2\cdot x^2\cdot J0(x) \qquad \delta_{aPh}(\theta_0) := -\frac{R_0^4}{L_t^4}\cdot\left(3\cdot\rho_{02}^2\cdot\theta_0^2 + 2\cdot H(\theta_0)\cdot\rho_{02}^2\cdot\cos(\psi_0 + 2\cdot\varphi_{02})\right)$$

$\mu_{aPh}(\theta_0) := -86400\cdot\delta_{aPh}(\theta_0)$ $\quad$ $\boxed{\mu_{aPh}(\theta_0) = 0.119}$ $\quad$ $\boxed{\mu_{aPh}(180\cdot deg) = 0.238}$

Modifications de la forme des courbes terminales

Valeurs de test $\quad$ $x_1 := 1.02$ $\quad$ $x_2 := 0.98$ $\quad$ $r_t := x_1\cdot r_{Ph}$ $\quad$ $r_{t'} := x_2\cdot r_{Ph}$

$\rho_1(r_t) := |Z_1(r_t)|$ $\quad$ $\varphi_1(r_t) := \arg(Z_1(r_t))$ $\quad$ $\rho_2(r_t) := |Z_2(r_t)|$ $\quad$ $\varphi_2(r_t) := \arg(Z_2(r_t))$

$\boxed{\rho_1(r_t) = 0.106}$ $\quad$ $\boxed{\varphi_1(r_t) = 120.021\,deg}$ $\quad$ $\boxed{\rho_2(r_t) = 1.346}$ $\quad$ $\boxed{\varphi_2(r_t) = 142.261\,deg}$

$\rho'_1(r_{t'}) := |Z_1(r_{t'})|$ $\quad$ $\varphi'_1(r_{t'}) := \arg(Z_1(r_{t'}))$ $\quad$ $\rho'_2(r_{t'}) := |Z_2(r_{t'})|$ $\quad$ $\varphi'_2(r_{t'}) := \arg(Z_2(r_{t'}))$

$\boxed{\rho'_1(r_{t'}) = 0.108}$ $\quad$ $\boxed{\varphi'_1(r_{t'}) = -57.554\,deg}$ $\quad$ $\boxed{\rho'_2(r_{t'}) = 0.799}$ $\quad$ $\boxed{\varphi'_2(r_{t'}) = 150.556\,deg}$

Calcul numérique de la perturbation de période

$$\mathbf{w_t}(r_t,\theta) := \frac{\theta}{L_t}\cdot\left(i\cdot R_0\cdot\rho_1(r_t)\cdot e^{-i\cdot\varphi_1(r_t)} + \frac{\theta}{L_t}\cdot R_0^2\cdot\rho_2(r_t)\cdot e^{-i\cdot\varphi_2(r_t)}\right)\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\mathbf{OA}$$

$$\mathbf{w_{t'}}(r_{t'},\theta) := \frac{\theta}{L_t}\cdot\left(i\cdot R_0\cdot\rho'_1(r_{t'})\cdot e^{i\cdot\varphi'_1(r_{t'})} - \frac{\theta}{L_t}\cdot R_0^2\cdot\rho'_2(r_{t'})\cdot e^{i\cdot\varphi'_2(r_{t'})}\right)\cdot\mathbf{OB}\cdot e^{i\cdot\theta\cdot\frac{l_t+L}{L_t}}$$

$\mathbf{w}(r_t,r_{t'},\theta) := \mathbf{w_t}(r_t,\theta) + \mathbf{w_{t'}}(r_{t'},\theta)$ $\quad$ $\mathbf{w}(r_t,r_{t'},\theta_0) = -0.053 + 0.061i\,mm$

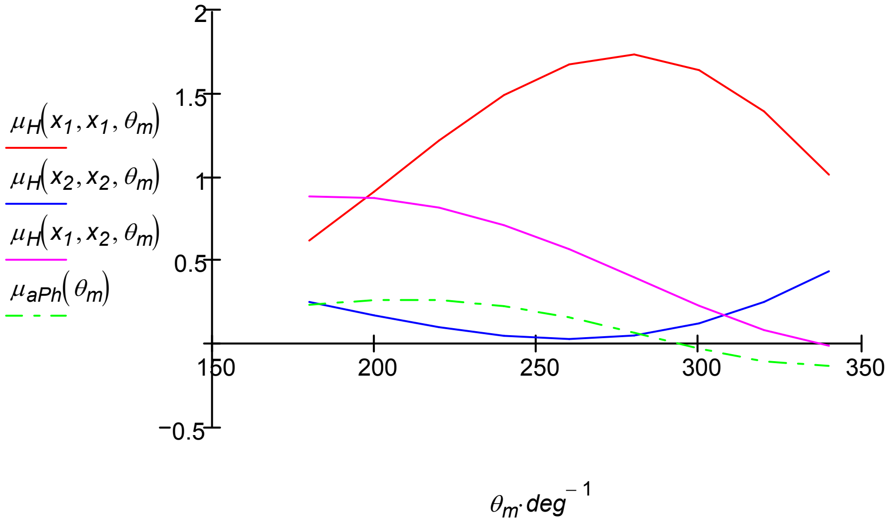
$\sigma2 := 24.144\cdot mm^2$

$$X(r_t, r_{t'}, \theta) := \frac{\left(\left|\mathbf{w}(r_t, r_{t'}, \theta)\right|\right)^2}{\sigma 2} \qquad \gamma(r_t, r_{t'}, \theta) := \frac{d}{d\theta}X(r_t, r_{t'}, \theta)$$

$$\delta_H(r_t, r_{t'}, \theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma(r_t, r_{t'}, \theta_0\cdot cos(\varphi))\cdot cos(\varphi)\,d\varphi \qquad x_1 := 1.02 \qquad x_2 := 0.98$$

$$\mu_H(x, x', \theta_0) := -86400\cdot\delta_H(x\cdot r_{Ph}, x'\cdot r_{Ph}, \theta_0) \qquad \boxed{\mu_H(x_1, x_2, \theta_0) = 0.489} \qquad \boxed{\mu_H(x_1, x_2, 180\cdot deg) = 0.889}$$

$$\theta_m := 180\cdot deg, 200\cdot deg .. 350\cdot deg$$



Solution analytique

$$A(r_t, r_{t'}) := \frac{R_0^{\,4}}{L_t^{\,2}}\cdot\left(\rho_1(r_t)^2 + \rho'_1(r_{t'})^2\right) \qquad B(r_t, r_{t'}) := \frac{3}{2}\cdot\frac{R_0^{\,6}}{L_t^{\,4}}\cdot\left(\rho_2(r_t)^2 + \rho'_2(r_{t'})^2\right)$$

$$C(r_t, r_{t'}) := 2\cdot\frac{R_0^{\,4}}{L_t^{\,2}}\cdot\rho_1(r_t)\cdot\rho'_1(r_{t'})\cdot cos\left(\psi_0 + \varphi_1(r_t) + \varphi'_1(r_{t'})\right)$$

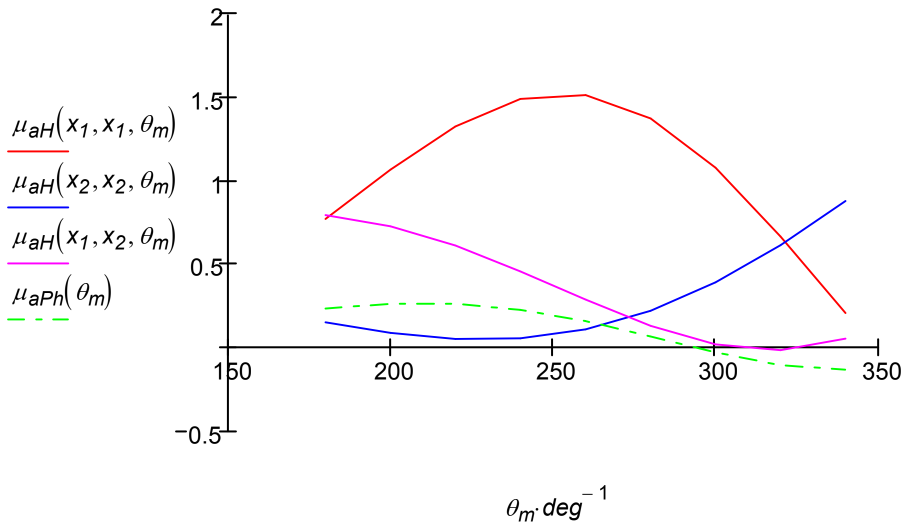
$$D(r_t, r_{t'}) := 2\cdot\frac{R_0^{\,5}}{L_t^{\,3}}\cdot\left(\rho_1(r_t)\cdot\rho'_2(r_{t'})\cdot cos\left(\psi_0 + \varphi_1(r_t) + \varphi'_2(r_{t'})\right) + \rho'_1(r_{t'})\cdot\rho_2(r_t)\cdot cos\left(\psi_0 + \varphi'_1(r_{t'}) + \varphi_2(r_t)\right)\right)$$

$$K(r_t, r_{t'}) := 2\cdot\frac{R_0^{\,6}}{L_t^{\,4}}\cdot\rho_2(r_t)\cdot\rho'_2(r_{t'})\cdot cos\left(\psi_0 + \varphi_2(r_t) + \varphi'_2(r_{t'})\right)$$

$$F(x) := J0(x) - x\cdot J1(x) \qquad G(x) := -x\cdot(J1(x) + x\cdot J0(x))$$

$$\delta_{aH}(r_t, r_{t'}, \theta_0) := \frac{-1}{R_0^{\,2}}\cdot\left(A(r_t, r_{t'}) + B(r_t, r_{t'})\cdot\theta_0^{\,2} + C(r_t, r_{t'})\cdot F(\theta_0) + D(r_t, r_{t'})\cdot G(\theta_0) + K(r_t, r_{t'})\cdot H(\theta_0)\right)$$

$$\mu_{aH}(x, x', \theta_0) := -86400\cdot\delta_{aH}(x\cdot r_{Ph}, x'\cdot r_{Ph}, \theta_0) \qquad \boxed{\mu_{aH}(x_1, x_2, \theta_0) = 0.21} \qquad \boxed{\mu_{aH}(x_1, x_2, 180\cdot deg) = 0.8}$$



Déplacement du centre de gravité

Cas de courbes de Phillips symétriques

$$\zeta_{aPh}(\theta) := \frac{\theta}{2 \cdot \psi_0^{\,2}} \cdot \rho_{02} \cdot \mathbf{OA} \cdot \left[-(4 \cdot i + \theta) \cdot e^{-i \cdot \varphi_{02}} + (4 \cdot i - \theta) \cdot e^{i \cdot (\psi_0 + \varphi_{02} + \theta)} \right]$$

Modifications de la forme des courbes terminales

$\kappa := 0.5$ $\kappa' := 1 - \kappa$

$$f_g(\theta, s) := \left[1 + i \cdot \theta \cdot \left(\frac{s}{L_t} - \kappa \right) \right] \cdot e^{i \cdot \theta \cdot \frac{s}{L_t}} \qquad f'_g(\theta, s) := i \cdot \frac{\theta}{L_t} \cdot \left[2 + i \cdot \theta \cdot \left(\frac{s}{L_t} - \kappa \right) \right] \cdot e^{i \cdot \theta \cdot \frac{s}{L_t}}$$

$$f_{gA}(\theta) := f_g(\theta, l_t) \qquad f_{gB}(\theta) := f_g(\theta, l_t + L) \qquad f'_{gA}(\theta) := f'_g(\theta, l_t) \qquad f'_{gB}(\theta) := f'_g(\theta, l_t + L)$$

$$\zeta_t(r_t, \theta) := \frac{1}{L_t} \cdot \left(R_0 \cdot \rho_1(r_t) \cdot e^{-i \cdot \varphi_1(r_t)} \cdot f_{gA}(\theta) - R_0^{\,2} \cdot \rho_2(r_t) \cdot e^{-i \cdot \varphi_2(r_t)} \cdot f'_{gA}(\theta) \right) \cdot \mathbf{OA}$$

$$\zeta_{t'}(r_{t'}, \theta) := \frac{1}{L_t} \cdot \left(R_0 \cdot \rho'_1(r_{t'}) \cdot e^{i \cdot \varphi'_1(r_{t'})} \cdot f_{gB}(\theta) + R_0^{\,2} \cdot \rho'_2(r_{t'}) \cdot e^{i \cdot \varphi'_2(r_{t'})} \cdot f'_{gB}(\theta) \right) \cdot \mathbf{OB}$$

$$\zeta(r_t, r_{t'}, \theta) := \zeta_t(r_t, \theta) + \zeta_{t'}(r_{t'}, \theta) \qquad \zeta(r_t, r_{t'}, \theta_0) = 0.011 + 5.459i \times 10^{-3}\, mm$$

Approximation de Haag

$$\zeta_{at}(r_t, \theta) := \frac{1}{L_t} \cdot \left[R_0 \cdot \rho_1(r_t) \cdot e^{-i \cdot \varphi_1(r_t)} \cdot (1 - i \cdot \kappa \cdot \theta) - \frac{\theta}{L_t} \cdot R_0^{\,2} \cdot \rho_2(r_t) \cdot e^{-i \cdot \varphi_2(r_t)} \cdot (2 \cdot i + \kappa \cdot \theta) \right] \cdot \mathbf{OA}$$

$$\zeta_{at'}(r_{t'}, \theta) := \frac{1}{L_t} \cdot \left[R_0 \cdot \rho'_1(r_{t'}) \cdot e^{i \cdot \varphi'_1(r_{t'})} \cdot (1 + i \cdot \kappa' \cdot \theta) + \frac{\theta}{L_t} \cdot R_0^{\,2} \cdot \rho'_2(r_{t'}) \cdot e^{i \cdot \varphi'_2(r_{t'})} \cdot (2 \cdot i - \kappa' \cdot \theta) \right] \cdot \mathbf{OB} \cdot e^{i \cdot \theta}$$

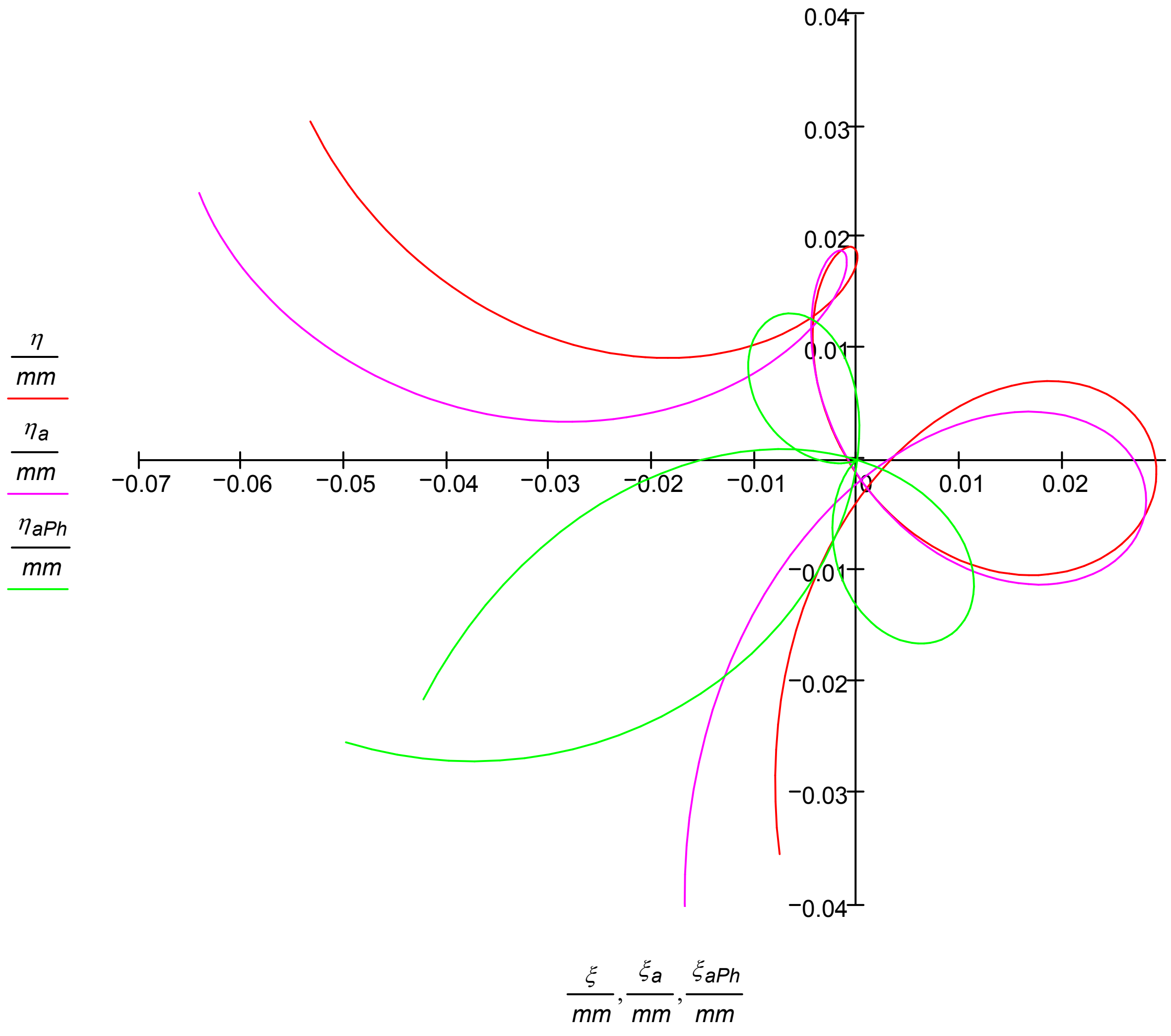
$$\zeta_a(r_t, r_{t'}, \theta) := \zeta_{at}(r_t, \theta) + \zeta_{at'}(r_{t'}, \theta)$$

Graphes du déplacement du centre de gravité

$n := 201 \qquad i := 0 .. n-1 \qquad \Delta\theta := \dfrac{4\cdot\pi}{n-1} \qquad \theta_i := -2\cdot\pi + i\cdot\Delta\theta$

$\xi_{aPh_i} := Re\left(\zeta_{aPh}(\theta_i)\right) \qquad \eta_{aPh_i} := Im\left(\zeta_{aPh}(\theta_i)\right)$

$\xi_{a_i} := Re\left(\zeta_a(r_t, r_{t'}, \theta_i)\right) \qquad \eta_{a_i} := Im\left(\zeta_a(r_t, r_{t'}, \theta_i)\right) \qquad \xi_i := Re\left(\zeta(r_t, r_{t'}, \theta_i)\right) \qquad \eta_i := Im\left(\zeta(r_t, r_{t'}, \theta_i)\right)$



Perturbation de période en position verticale

Cas de courbes de Phillips symétriques

$Q(\theta_0) := 5\cdot J0(\theta_0) - \theta_0\cdot J1(\theta_0)$

$Z_{aPh}(\theta_0) := \dfrac{-R_0^{\,2}}{2\cdot L_t^{\,2}}\cdot\rho_{02}\cdot\left(\mathbf{OA}\cdot e^{-i\cdot\varphi_{02}} + Q(\theta_0)\cdot\mathbf{OB}\cdot e^{i\cdot\varphi_{02}}\right) \qquad \delta_{VPh}(\theta_0) := -g\cdot\dfrac{m_s\cdot L}{E\cdot I_{33}}\cdot Im\left(Z_{aPh}(\theta_0)\right)$

$\mu_{VPh}(\theta_0) := -86400\cdot\delta_{VPh}(\theta_0) \qquad \boxed{\mu_{VPh}(\theta_0) = -1.832} \qquad \boxed{\mu_{VPh}(180\cdot deg) = 0.806}$

Modifications de la forme des courbes terminales

$$f(\theta_0, s) := \frac{s}{L_t} \cdot \left[\left(\kappa - \frac{s}{L_t} \right) \cdot J0\left(\theta_0 \cdot \frac{s}{L_t} \right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s}{L_t} \right) \right] \qquad f'(\theta_0, s) := \frac{d}{ds} f(\theta_0, s)$$

$$f_A(\theta_0) := f(\theta_0, l_t) \qquad f_B(\theta_0) := f(\theta_0, l_t + L) \qquad f'_A(\theta_0) := f'(\theta_0, l_t) \qquad f'_B(\theta_0) := f'(\theta_0, l_t + L)$$

$$Z_t(r_t, \theta) := \frac{1}{L_t} \cdot \left(R_0 \cdot \rho_1(r_t) \cdot e^{-i \cdot \varphi_1(r_t)} \cdot f_A(\theta) - R_0^{\,2} \cdot \rho_2(r_t) \cdot e^{-i \cdot \varphi_2(r_t)} \cdot f'_A(\theta) \right) \cdot \mathbf{OA}$$

$$Z_{t'}(r_{t'}, \theta) := \frac{1}{L_t} \cdot \left(R_0 \cdot \rho'_1(r_{t'}) \cdot e^{i \cdot \varphi'_1(r_{t'})} \cdot f_B(\theta) + R_0^{\,2} \cdot \rho'_2(r_{t'}) \cdot e^{i \cdot \varphi'_2(r_{t'})} \cdot f'_B(\theta) \right) \cdot \mathbf{OB}$$

$$Z_a(r_t, r_{t'}, \theta) := Z_t(r_t, \theta) + Z_{t'}(r_{t'}, \theta) \qquad \delta_V(r_t, r_{t'}, \theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_a(r_t, r_{t'}, \theta_0))$$

$$\mu_V(x, x', \theta_0) := -86400 \cdot \delta_V(x \cdot r_{Ph}, x' \cdot r_{Ph}, \theta_0) \qquad \boxed{\mu_V(x_1, x_2, \theta_0) = 0.296} \qquad \boxed{\mu_V(x_1, x_2, 180 \cdot deg) = 1.256}$$

Approximation de Haag

$$F_1(x) := (1 - \kappa) \cdot J0(x) + \frac{1}{x} \cdot J1(x) \qquad F_2(x) := 2 \cdot J0(x) + (1 - \kappa) \cdot F(x)$$

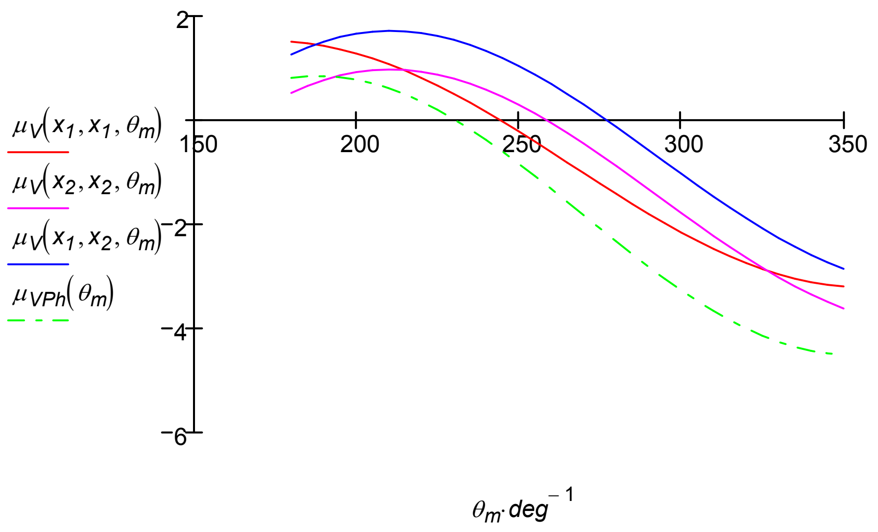
$$Z_{at}(r_t, \theta_0) := \frac{\kappa}{L_t^{\,2}} \cdot \left(l_t \cdot R_0 \cdot \rho_1(r_t) \cdot e^{-i \cdot \varphi_1(r_t)} - R_0^{\,2} \cdot \rho_2(r_t) \cdot e^{-i \cdot \varphi_2(r_t)} \right) \cdot \mathbf{OA}$$

$$Z_{at'}(r_{t'}, \theta_0) := \frac{-1}{L_t} \cdot \left(R_0 \cdot \rho'_1(r_{t'}) \cdot e^{i \cdot \varphi'_1(r_{t'})} \cdot F_1(\theta_0) + \frac{R_0^{\,2}}{L_t} \cdot \rho'_2(r_{t'}) \cdot e^{i \cdot \varphi'_2(r_{t'})} \cdot F_2(\theta_0) \right) \cdot \mathbf{OB}$$

$$Z_{ah}(r_t, r_{t'}, \theta_0) := Z_{at}(r_t, \theta_0) + Z_{at'}(r_{t'}, \theta_0) \qquad \delta_{aV}(r_t, r_{t'}, \theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_{ah}(r_t, r_{t'}, \theta_0))$$

$$\mu_{aV}(x, x', \theta_0) := -86400 \cdot \delta_{aV}(x \cdot r_{Ph}, x' \cdot r_{Ph}, \theta_0) \qquad \boxed{\mu_{aV}(x_1, x_2, \theta_0) = -0.971} \qquad \boxed{\mu_{aV}(x_1, x_2, 180 \cdot deg) = 1.079}$$

$$\theta_m := 180 \cdot deg, 185 \cdot deg .. 350 \cdot deg$$



Test des approximations de Haag